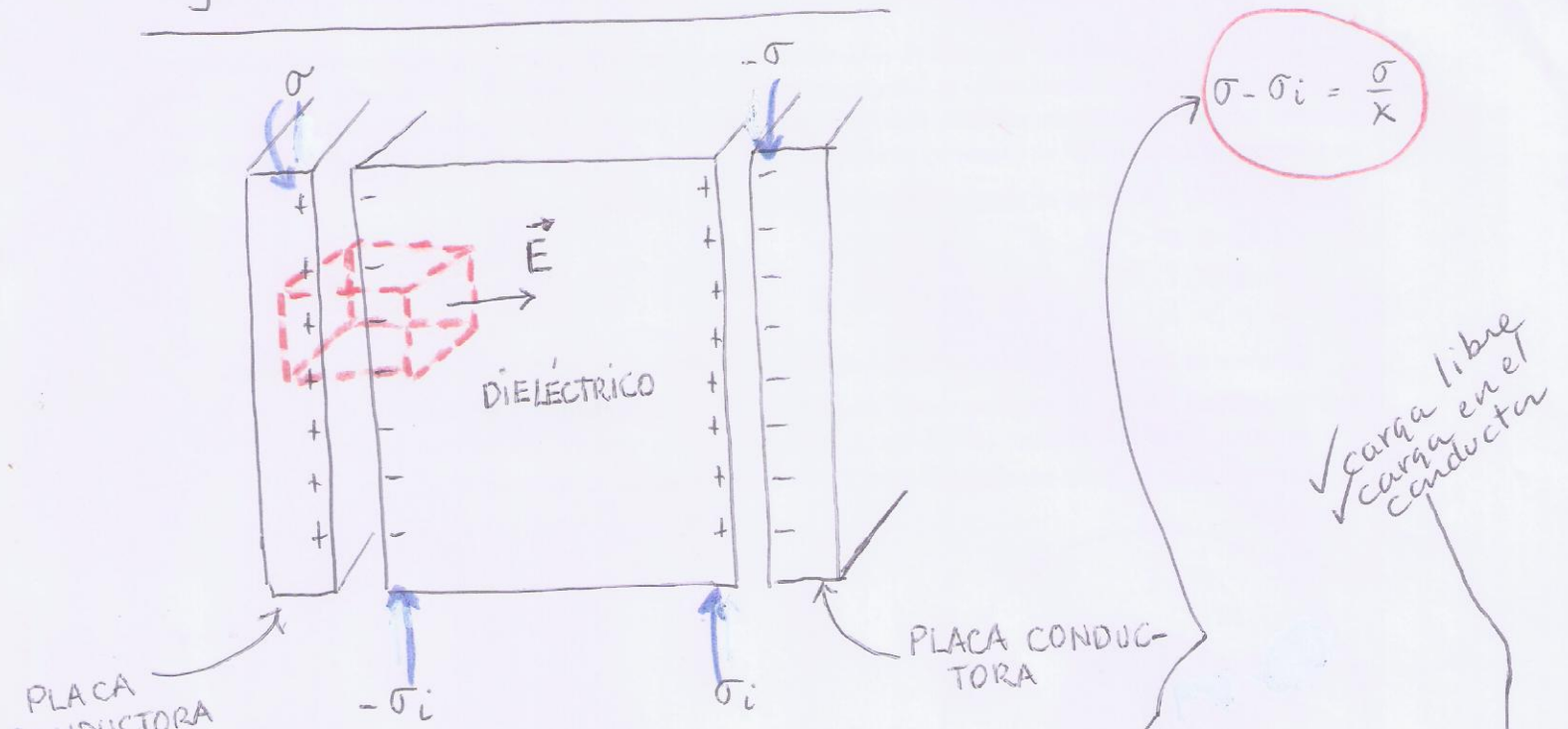
Ley de Gauss en dieléctricos



$$\oint_{SUP} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A - \sigma_i A}{\varepsilon_0} = \frac{(\sigma - \sigma_i)A}{\varepsilon_0} = \frac{(\sigma/\kappa)A}{\varepsilon_0} = \frac{Q}{\kappa \varepsilon_0}$$

Q_{libre} = carga en la placa conductora

$$\oint_{SUP} \vec{E} \cdot d\vec{A} = \frac{Q_{libre}}{\kappa \varepsilon_0}$$

$$\oint_{SUP} (\kappa \varepsilon_0 \vec{E}) \cdot d\vec{A} = Q_{libre}$$

$$\oint_{SUP} \vec{D} \cdot d\vec{A} = Q_{libre}$$

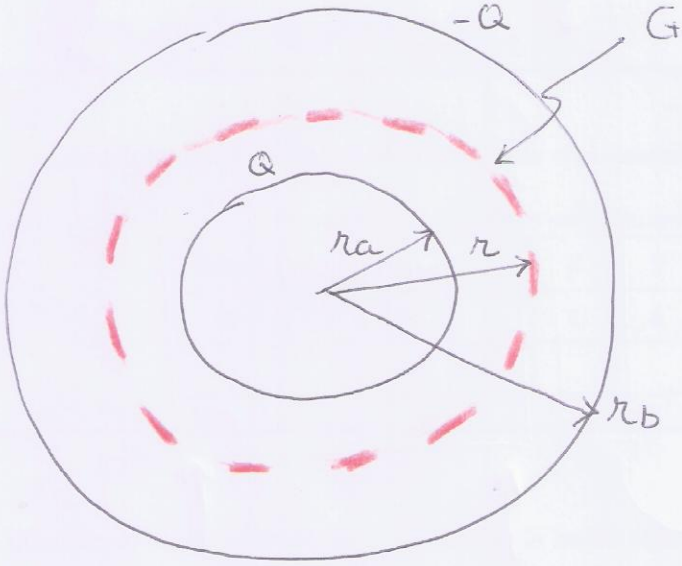
$\vec{D} = \kappa \varepsilon_0 \vec{E}$ = Desplazamiento eléctrico

$\vec{D} = \varepsilon \vec{E}$ $\qquad \varepsilon = \kappa \varepsilon_0$

ε = permitividad del dieléctrico

ε_0 = permitividad del vacío

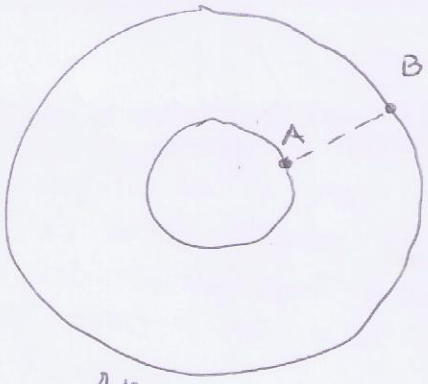
Condensador esférico con dieléctrico



Gauss $\Rightarrow E\,4\pi r^2 = \frac{Q}{\varepsilon_0}$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$C_0 = ?$ ← Sin dieléctrico



$$V_A - V_B = \int_{r_a}^{r_b} \vec{E}\cdot d\vec{r} = \int_{r_a}^{r_b} E\,dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,dr$$

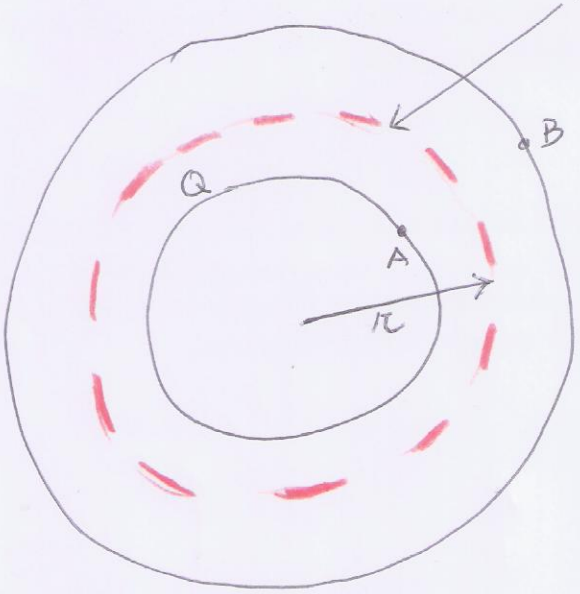
$$V_A - V_B = \frac{1}{4\pi\varepsilon_0} Q \left(-\frac{1}{r}\right)\Big|_{r_a}^{r_b} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\frac{r_b - r_a}{r_a r_b}$$

$$C_0 = \frac{Q}{V_A - V_B} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\frac{(r_b - r_a)}{r_a r_b}}$$

$$C_0 = 4\pi\varepsilon_0 \frac{r_a r_b}{r_b - r_a}$$

Con dieléctrico



$$\oint_{SUP} \vec{D} \cdot d\vec{A} = Q_{libre} = Q \quad \text{(encerrada)}$$

$$\oint_{SUP} (\kappa \varepsilon_0 \vec{E}) \cdot d\vec{A} = \kappa \varepsilon_0 E 4\pi r^2$$

$$E = \frac{Q}{\kappa \varepsilon_0 4\pi r^2}$$

$$E = \frac{1}{4\pi(\kappa\varepsilon_0)} \frac{Q}{r^2}$$

$$E = \frac{1}{4\pi\varepsilon} \frac{Q}{r^2}$$

La capacidad es:

$$C' = \frac{Q}{V'_{AB}}$$

$$V'_{AB} = \int_{r_a}^{r_b} E\, dr$$

$$\Rightarrow \quad V'_{AB} = \frac{Q}{4\pi\kappa\varepsilon_0} \left(\frac{r_b - r_a}{r_a r_b} \right)$$

$$C' = 4\pi\kappa\varepsilon_0 \frac{r_a r_b}{r_b - r_a}$$

$$C' = \kappa\, C_0$$

$\Rightarrow C' > C_0$ porque $\kappa > 1$

Al introducir el dieléctrico, aumenta la capacidad